Improved Control Charts for Attributes

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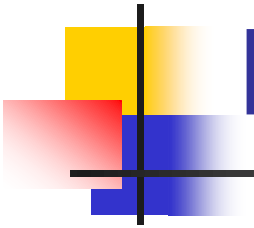


Key Points

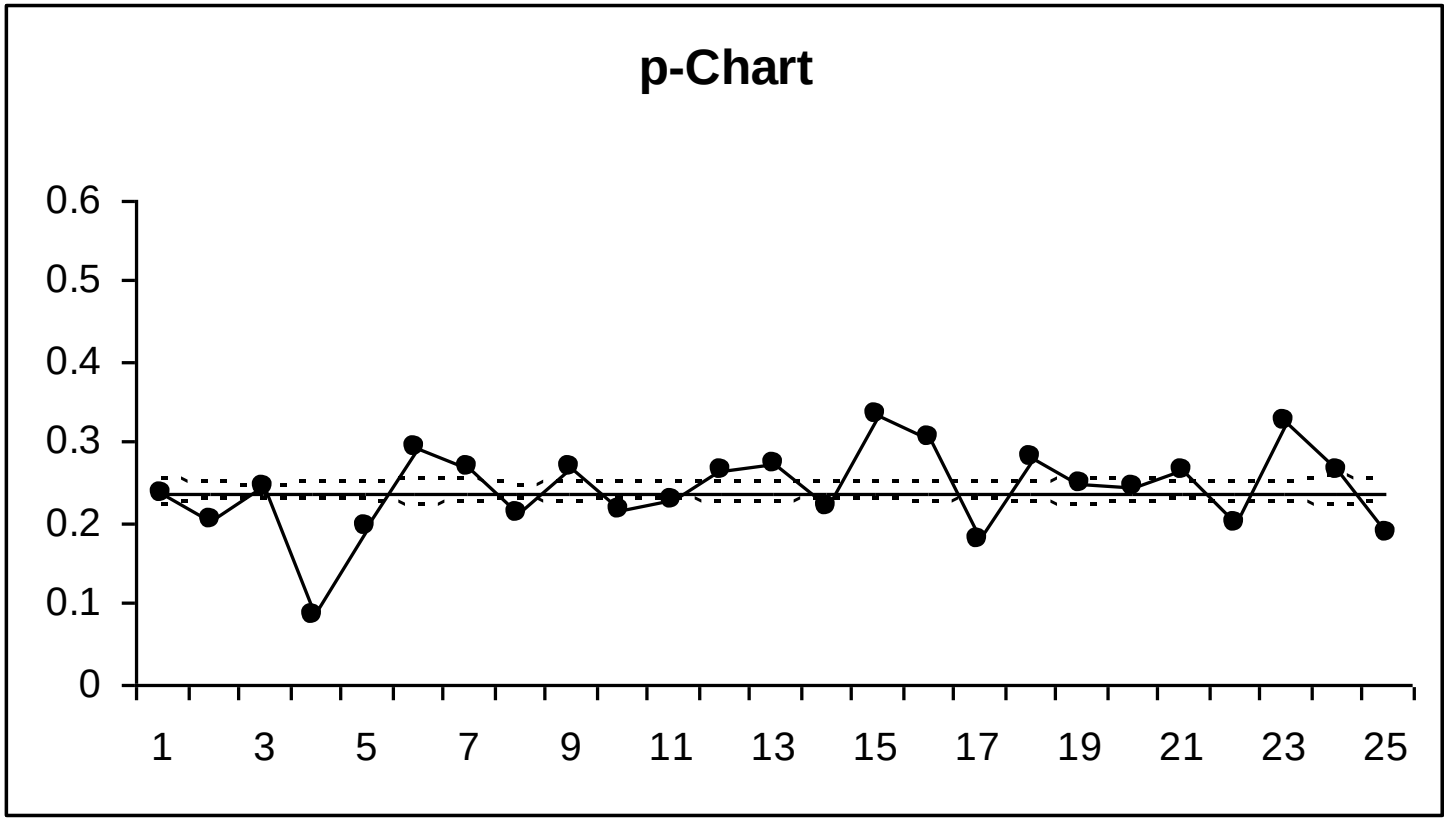
- p-charts and u-charts are often wrong
- Too many false alarms
- Why this happens
- Traditional remedy
- Better ways

The Classical p-Chart

$$\sigma_{p_i} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

$$UCL(p_i) = \bar{p} + 3\sigma_{p_i}$$

The Classical p-Chart



Why does this happen?

- The "binomial assumption" is not true;
- The *parameter* is changing over time;
- There is *common cause* variation here that cannot be explained by "intra-subgroup" sampling variation alone;
- The Western Electric Handbook (1956) gave us a fix…

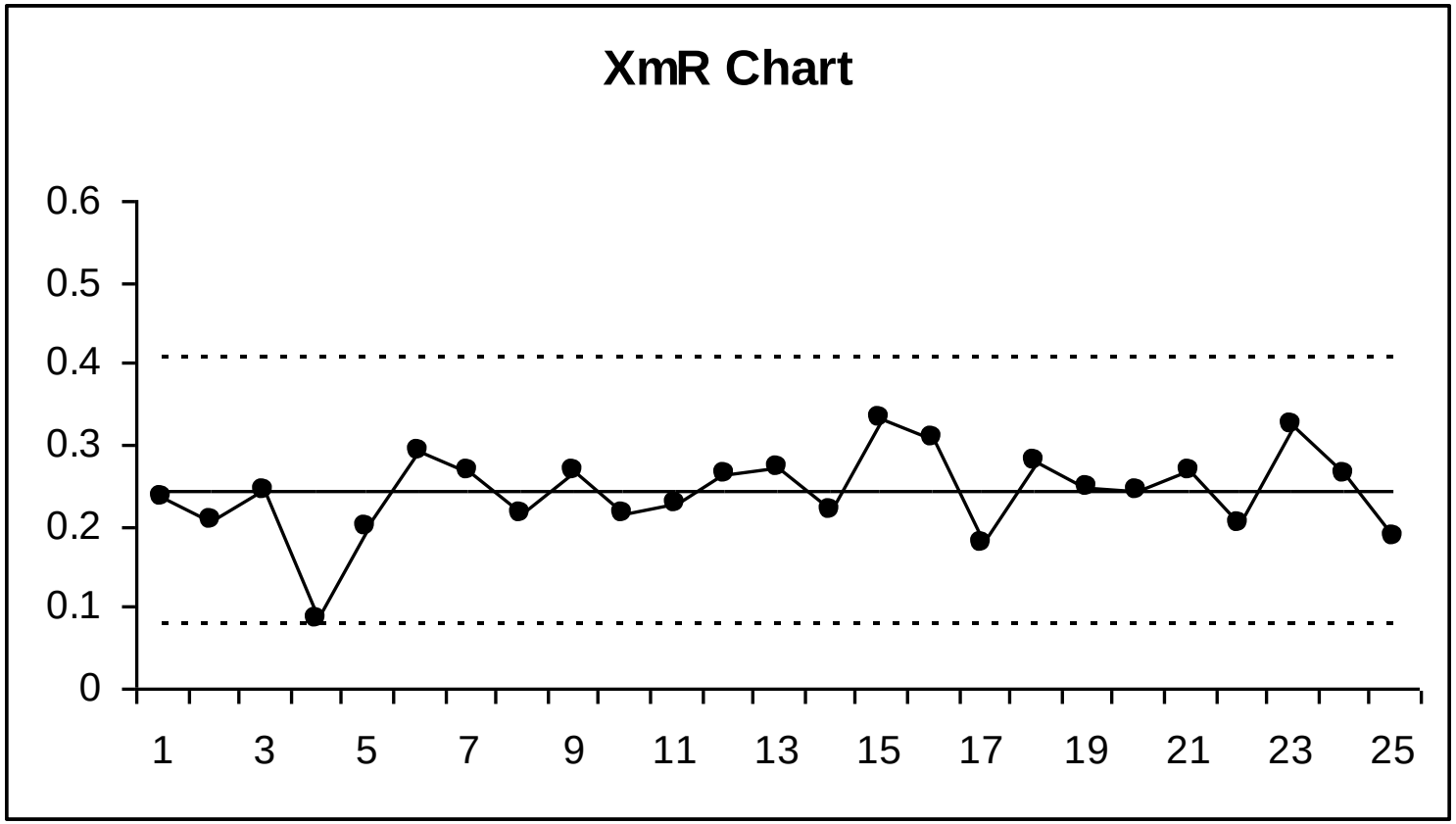
The Individuals (XmR) Chart

$$x_i = p_i$$

$$\sigma_x = \frac{1}{k-1}\sum_{i=2}^{k}\left|x_i - x_{i-1}\right| \div 1.128$$

$$UCL(x) = \bar{p} + 3\sigma_x$$

The Individuals (XmR) Chart



So, what's wrong with it?

- If the subgroup sizes vary, this is biased
- For example, the average sample size here is 12,429. For point #4, it is larger than that: 15,122
- If the control limits "wiggled" to reflect varying sampling error, might #4 be out of control?

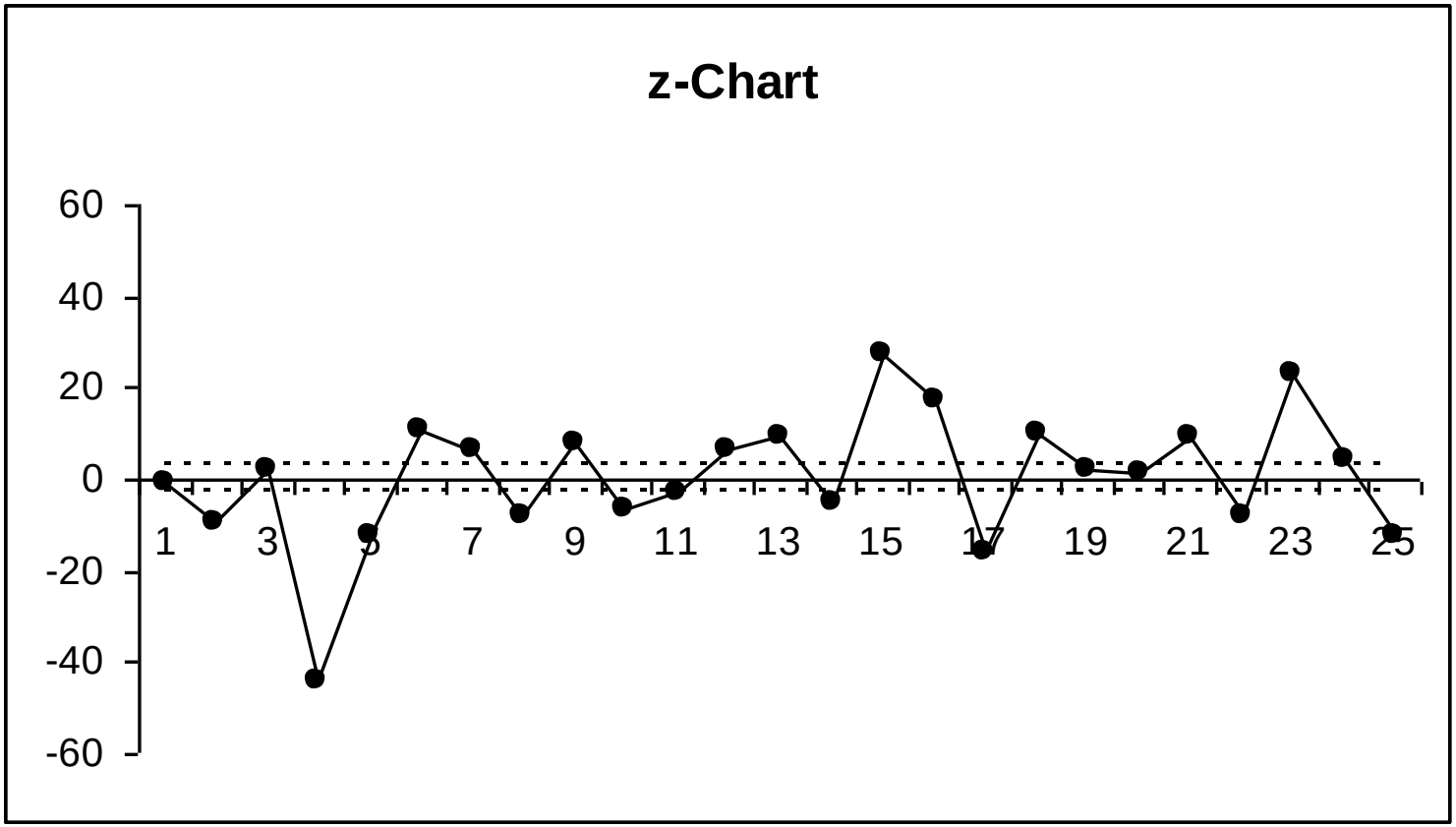
The z-Chart

$$z_i = \frac{p_i - \bar{p}}{\sigma_{p_i}}$$

$$\sigma_{z_i} = 1 \textbf{ (by assumption)}$$

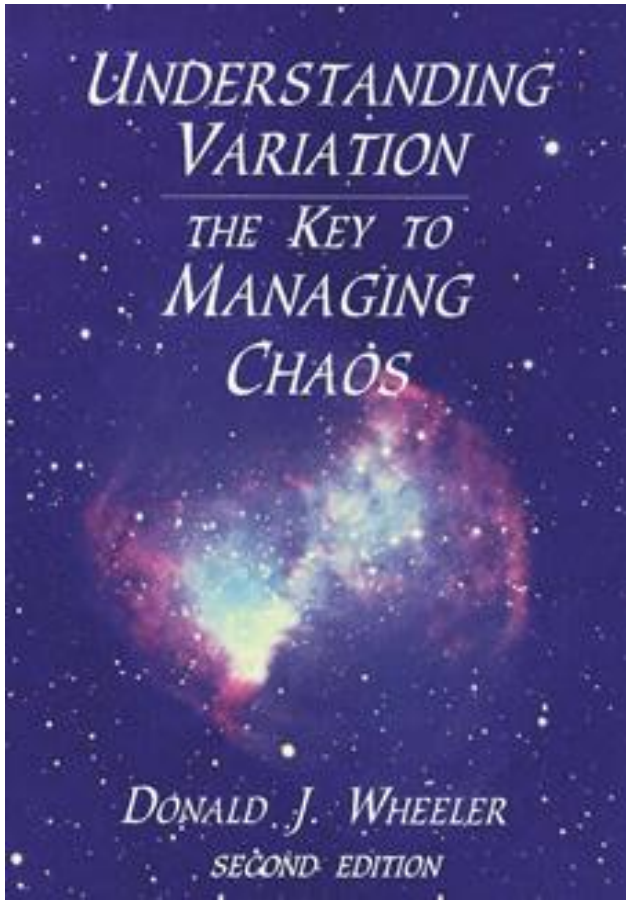
$$UCL = 3\sigma_{z_i} = 3$$

The z-Chart



According to Don Wheeler ...

"Why assume the variation when you can measure it?"



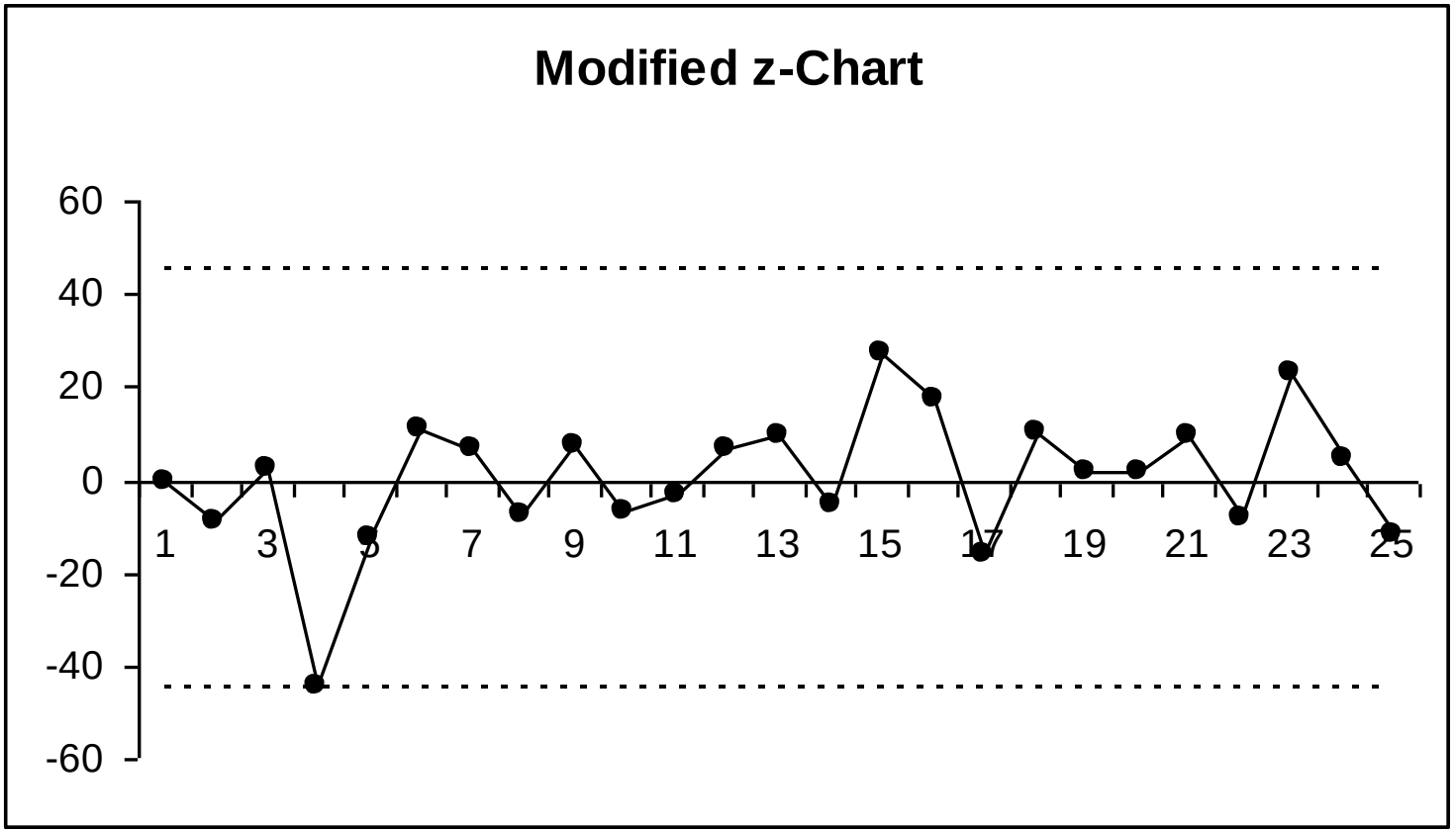
The Modified z-Chart

$$z_i = \frac{p_i - \bar{p}}{\sigma_{p_i}}$$

$$\sigma_z = \frac{1}{k-1}\sum_{i=2}^{k}\left|z_i - z_{i-1}\right| \div 1.128$$

$$UCL(z) = 3\sigma_z$$

The Modified z-Chart



What is σ_z?

- It is the relative amount of variation not explained by the binomial sampling variation within subgroups.
- As in any Individuals Chart, this is still "common cause" variation.
- We have merely redefined the "rational subgroup" for this situation.

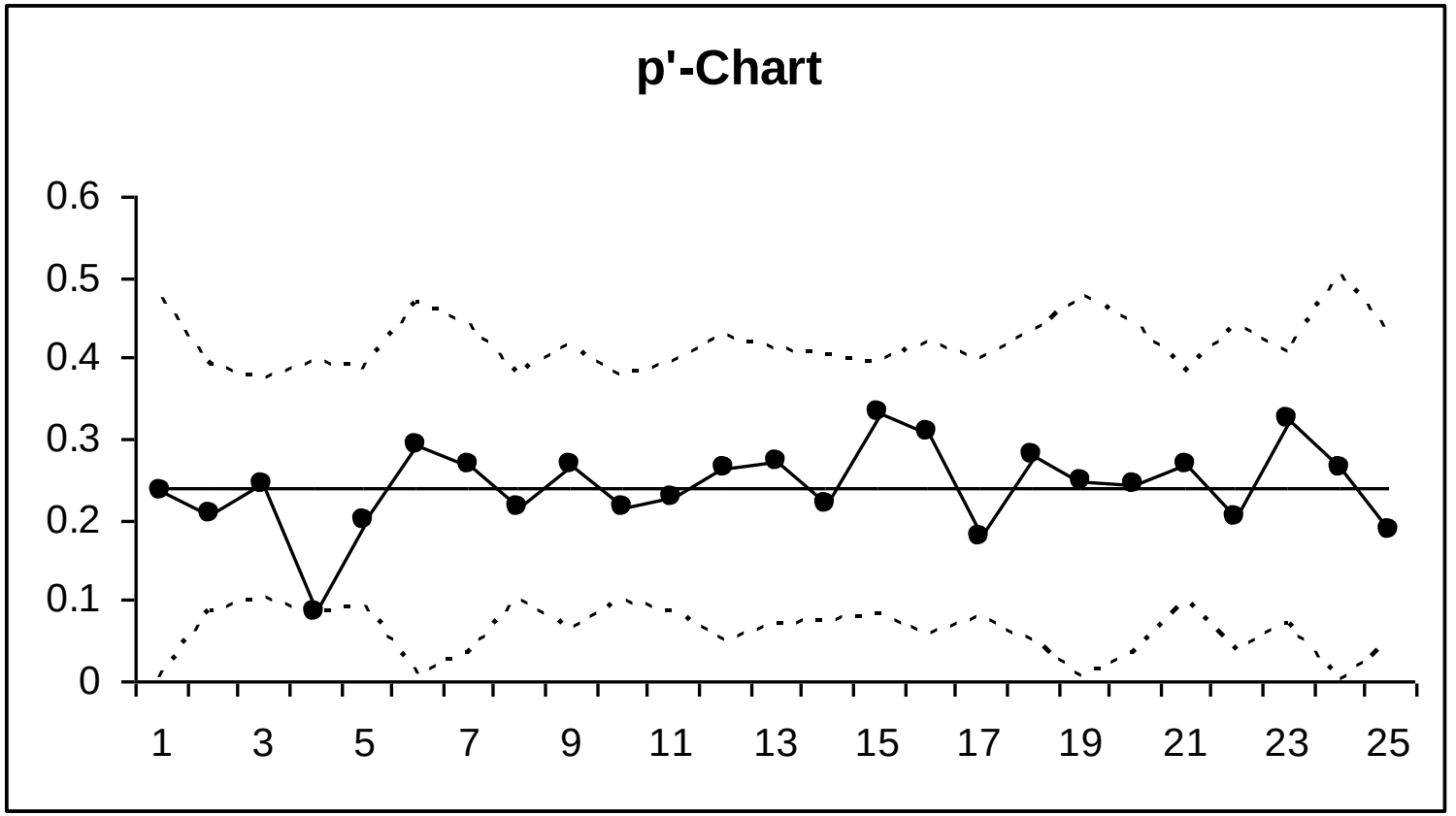
Laney's p'-Chart

$$p'_i = \bar{p} + \sigma_{p_i} z_i$$

$$\sigma_{p'_i} = \sigma_{p_i} \sigma_z$$

$$UCL(p'_i) = \bar{p} + 3\sigma_{p_i} \sigma_z$$

Laney's p'-Chart

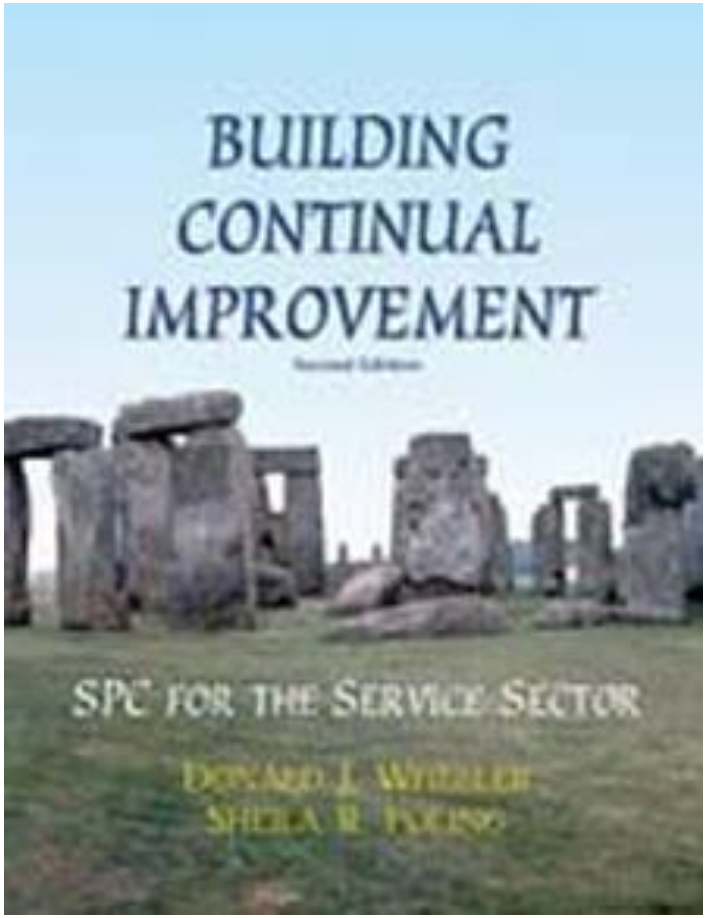


Some observations

- If the data are binomial, this becomes the p-chart
- If the subgroup sizes are all the same, this becomes the XmR chart.

Wait! There's more…

Wheeler again:



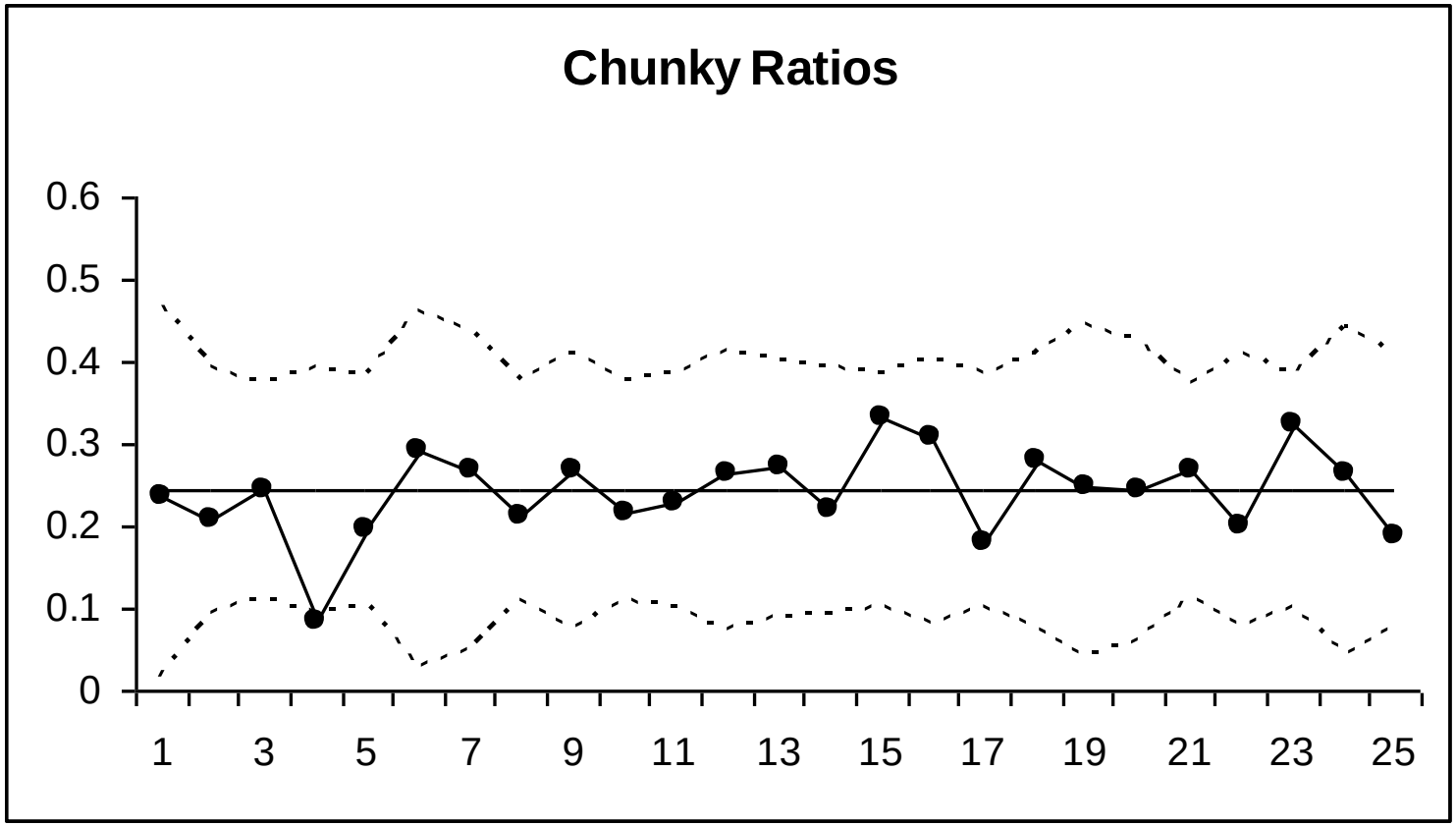
Wheeler's "Chunky Ratios"

$$x_i = p_i$$

$$\sigma_x = \frac{1}{k-1}\sum_{i=2}^{k}\left|x_i - x_{i-1}\right| \div 1.128$$

$$UCL(x) = \bar{p} + 3\sigma_x\sqrt{\frac{\bar{n}}{n_i}}$$

Wheeler's "Chunky Ratios"

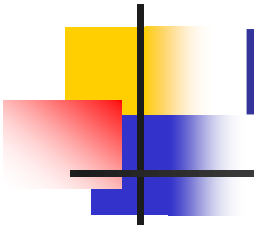


Laney vs. Wheeler

- Both methods give almost the same result;
- By equally weighting all subgroups, regardless of size, Wheeler's method may have some bias;
- Unquestionably, Wheeler's method is simpler.

So, Regis, is this the "Final Answer"?

- Not by a long shot!
- Christa Carter, PhD - University of Alabama, 2002:
  - Bayesian Approach: Beta Prior and Beta-Binomial (Negative Exponential) Posterior



Key Points

- p-charts and u-charts are often wrong
- Too many false alarms
- Why this happens
- Traditional remedy
- Better ways: The p’-chart; chunky ratios; Bayesian approach
- More research is needed